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Non-existence of New Quantum Ergosphere Effect of a Vaidya-type Black Hole

S. Q. Wu
Institute of Particle Physics, Hua-Zhong Normal University
Wuhan 430079, P.R. China
E-mail: sqwu@iopp.ccnu.edu.cn

X. Cai
Institute of Particle Physics, Hua-Zhong Normal University
Wuhan 430079, P.R. China
E-mail: xcai@ccnu.edu.cn

Abstract: Hawking evaporation of Dirac particles and scalar fields in a Vaidya-type black hole is investigated by the method of generalized tortoise coordinate transformation. It is shown that Hawking radiation of Dirac particles does not exist for P_1, Q_2 components but for P_2, Q_1 components in any Vaidya-type black holes. Both the location and the temperature of the event horizon change with time. The thermal radiation spectrum of Dirac particles is the same as that of Klein-Gordon particles. We demonstrates that there is no new quantum ergosphere effect in the thermal radiation of Dirac particles in any spherically symmetry black holes.

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1. Introduction

Hawking's investigation of quantum effects[1] interpreted as the emission of a thermal spectrum of particles by a black hole event horizon sets a remarkable landmark in black hole physics. During the last decade, the Hawking radiation of Dirac particles had been extensively investigated in some spherically symmetric and non-static black holes.[2] However, most of these studies concentrated on the spin state $p = 1/2$ of the four-component Dirac spinors. Recently, the Hawking radiation of Dirac particles of spin state $p = -1/2$ attracted a little more attention.[3,4] In several papers, Li et al.[3,4] declared that they had found a kind of new quantum thermal effect for the Vaidya-Bonner-de Sitter black hole. Basing upon the generalized Teukolsky-type master equation[5] for fields of spin ($s = 0, 1/2, 1$ and 2 for the scalar, Dirac, electromagnetic and gravitational field, respectively) in a Vaidya-type space-time, they showed that the massive Dirac field of spin state $p = 1/2$ differs greatly from that of spin state $p = -1/2$ in radiative mechanism, and suggested that it originate from the variance of Dirac vacuum near the event horizon in the non-static space-times

caused by spin state. Further, they conjectured that this effect originates from the quantum ergosphere,[6] that is, the quantum ergosphere can influence the radiative mechanism of a black hole. As far as their master equations[3,4] are concerned, their argument, in fact, sounds only for the massive spin-1/2 particles because only the mass of Klein-Gordon particle and that of Dirac particle are nonzero. An exotic feature of this effect is its obvious dependence on the mass μ_0 of spin-1/2 particles.

In this letter, we study the Hawking effect of Dirac and Klein-Gordon particles in a Vaidya-type black hole by means of the generalized tortoise transformation (GTCT) method. We consider simultaneously the limiting forms of the first-order and second-order forms of Dirac equation near the event horizon because the Dirac spinors should satisfy both of them. The event horizon equation, the Hawking temperature and the thermal radiation spectrum of electrons are in accord with others. We prove rigorously that the Hawking radiation takes place only for P_2, Q_1 but not for P_1, Q_2 components of Dirac spinors. The origin of this asymmetry of the Hawking radiation of different spinorial components probably stem from the asymmetry of space-time in the advanced Eddington-Finkelstein coordinate system. Besides, we point out that there could not have been any new quantum ergosphere effect in the Hawking radiation of Dirac particles in any spherically symmetric black hole whether it is static or non-static. This conclusion is contrary completely to that of Li's[3,4] who argued that the radiative mechanism of massive spin fields depends on the spin state.

The paper is outlined as follows: In section 2, the explicit form of Dirac equation in the Vaidya-type black hole is presented in spinorial formalism, the event horizon equation is derived in Sec. 3. Then we obtain the Hawking temperature and the thermal radiation spectrum in Sec. 4 and 5, respectively. The Hawking evaporation of Klein-Gordon particles is re-examined in Sec. 6. Finally we give some discussions.

2. Dirac equation

The metric of a Vaidya-type black hole with the cosmological constant Λ is given in the advanced Eddington-Finkelstein coordinate system by

$$ds^2 = 2dv(Gdv - dr) - r^2(d\theta^2 + \sin^2\theta d\varphi^2)\,, \tag{1}$$

where $2G = 1 - \frac{2M(v)}{r} - \frac{\Lambda}{3}r^4$, in which the mass M of the hole is a function of the advanced time v.

We establish such a complex null-tetrad $\{\boldsymbol{l}, \boldsymbol{n}, \boldsymbol{m}, \overline{\boldsymbol{m}}\}$ that satisfies the orthogonal conditions $\boldsymbol{l}\cdot\boldsymbol{n} = -\boldsymbol{m}\cdot\overline{\boldsymbol{m}} = 1$. Thus the covariant one-forms can be written as

$$\begin{aligned}
\boldsymbol{l} &= dv\,, & \boldsymbol{m} &= \frac{-r}{\sqrt{2}}\left(d\theta + i\sin\theta d\varphi\right)\,, \\
\boldsymbol{n} &= Gdv - dr\,, & \overline{\boldsymbol{m}} &= \frac{-r}{\sqrt{2}}\left(d\theta - i\sin\theta d\varphi\right)\,.
\end{aligned} \tag{2}$$

and their corresponding directional derivatives are

$$D = -\frac{\partial}{\partial r}\,, \qquad \delta = \frac{1}{\sqrt{2}r}\left(\frac{\partial}{\partial\theta} + \frac{i}{\sin\theta}\frac{\partial}{\partial\varphi}\right)\,,$$
$$\Delta = \frac{\partial}{\partial v} + G\frac{\partial}{\partial r}\,, \quad \overline{\delta} = \frac{1}{\sqrt{2}r}\left(\frac{\partial}{\partial\theta} - \frac{i}{\sin\theta}\frac{\partial}{\partial\varphi}\right)\,. \tag{3}$$

It is not difficult to compute the non-vanishing Newman-Penrose complex spin coefficients[7] in the above null-tetrad as follows

$$\mu = \frac{G}{r}\,, \quad \gamma = -\frac{G_{,r}}{2} = -\frac{dG}{2dr}\,, \quad \beta = -\alpha = \frac{\cot\theta}{2\sqrt{2}r}\,. \tag{4}$$

Inserting for the needed Newman-Penrose spin coefficients into the spinor form of the four coupled Chandrasekhar-Dirac equations[8] describing the dynamic behavior of spin-1/2 particles,

$$(D + \epsilon - \rho)F_1 + (\overline{\delta} + \tilde{\pi} - \alpha)F_2 = \frac{i\mu_0}{\sqrt{2}}G_1\,,$$
$$(\Delta + \mu - \gamma)F_2 + (\delta + \beta - \tau)F_1 = \frac{i\mu_0}{\sqrt{2}}G_2\,,$$
$$(D + \epsilon^* - \rho^*)G_2 - (\delta + \tilde{\pi}^* - \alpha^*)G_1 = \frac{i\mu_0}{\sqrt{2}}F_2\,,$$
$$(\Delta + \mu^* - \gamma^*)G_1 - (\overline{\delta} + \beta^* - \tau^*)G_2 = \frac{i\mu_0}{\sqrt{2}}F_1\,, \tag{5}$$

where μ_0 is the mass of Dirac particles, one arrives at

$$-\left(\frac{\partial}{\partial r} + \frac{1}{r}\right)F_1 + \frac{1}{\sqrt{2}r}\mathcal{L}_{1/2}F_2 = \frac{i\mu_0}{\sqrt{2}}G_1\,,$$
$$\frac{1}{2r^2}\mathcal{D}F_2 + \frac{1}{\sqrt{2}r}\mathcal{L}^\dagger{}_{1/2}F_1 = \frac{i\mu_0}{\sqrt{2}}G_2\,,$$
$$-\left(\frac{\partial}{\partial r} + \frac{1}{r}\right)G_2 - \frac{1}{\sqrt{2}r}\mathcal{L}^\dagger{}_{1/2}G_1 = \frac{i\mu_0}{\sqrt{2}}F_2\,,$$
$$\frac{1}{2r^2}\mathcal{D}G_1 - \frac{1}{\sqrt{2}r}\mathcal{L}_{1/2}G_2 = \frac{i\mu_0}{\sqrt{2}}F_1\,, \tag{6}$$

in which operators

$$\mathcal{D} = 2r^2\left(\frac{\partial}{\partial v} + G\frac{\partial}{\partial r}\right) + (r^2G)_{,r}\,,$$
$$\mathcal{L}_{1/2} = \frac{\partial}{\partial\theta} + \frac{1}{2}\cot\theta - \frac{i}{\sin\theta}\frac{\partial}{\partial\varphi}\,,$$
$$\mathcal{L}^\dagger{}_{1/2} = \frac{\partial}{\partial\theta} + \frac{1}{2}\cot\theta + \frac{i}{\sin\theta}\frac{\partial}{\partial\varphi}\,.$$

have been defined.

Further substitutions $P_1 = \sqrt{2}rF_1, P_2 = F_2, Q_1 = G_1, Q_2 = \sqrt{2}rQ_2$ yields

$$-\frac{\partial}{\partial r}P_1 + \mathcal{L}_{1/2}P_2 = i\mu_0 rQ_1\,, \quad \mathcal{D}P_2 + \mathcal{L}^{\dagger}{}_{1/2}P_1 = i\mu_0 rQ_2\,,$$
$$-\frac{\partial}{\partial r}Q_2 - \mathcal{L}^{\dagger}{}_{1/2}Q_1 = i\mu_0 rP_2\,, \quad \mathcal{D}Q_1 - \mathcal{L}_{1/2}Q_2 = i\mu_0 rP_1\,. \tag{7}$$

An apparent fact is that the Chandrasekhar-Dirac equation (7) could be satisfied by identifying Q_1, Q_2 with P_2^*, $-P_1^*$, respectively. So one may deal with a pair of components P_1, P_2 only.

3. Event Horizon

Now separating variables to Eq. (7) as

$$P_1 = R_1(v,r)S_1(\theta,\varphi)\,, \quad P_2 = R_2(v,r)S_2(\theta,\varphi)\,,$$
$$Q_1 = R_2(v,r)S_1(\theta,\varphi)\,, \quad Q_2 = R_1(v,r)S_2(\theta,\varphi)\,,$$

then we can decouple it to the radial part

$$\frac{\partial}{\partial r}R_1 = (\lambda - i\mu_0 r)R_2\,, \quad \mathcal{D}R_2 = (\lambda + i\mu_0 r)R_1\,, \tag{8}$$

and the angular part

$$\mathcal{L}^{\dagger}{}_{1/2}S_1 = -\lambda S_2\,, \quad \mathcal{L}_{1/2}S_2 = \lambda S_1\,, \tag{9}$$

where $\lambda = \ell + 1/2$ is a separation constant. Both functions $S_1(\theta,\varphi)$ and $S_2(\theta,\varphi)$ are, respectively, spinorial spherical harmonics ${}_sY_{\ell m}(\theta,\varphi)$ with spin-weight $s = \pm 1/2$ satisfying[9]

$$\Big[\frac{\partial^2}{\partial\theta^2} + \cot\theta\frac{\partial}{\partial\theta} + \frac{1}{\sin^2\theta}\frac{\partial^2}{\partial\varphi^2} + \frac{2is\cos\theta}{\sin^2\theta}\frac{\partial}{\partial\varphi}$$
$$- s^2\cot^2\theta + s + (\ell - s)(\ell + s + 1)\Big]{}_sY_{\ell m}(\theta,\varphi) = 0\,. \tag{10}$$

As to the thermal radiation, one should be concerned about the behavior of the radial part of Eq. (8) near the horizon only. Because the Vaidya-type black hole is spherically symmetric, one can introduce as a working ansatz the generalized tortoise coordinate transformation[10] as follows

$$r_* = r + \frac{1}{2\kappa}\ln[r - r_H(v)]\,, \quad v_* = v - v_0\,, \tag{11}$$

where $r_H = r_H(v)$ is the location of the event horizon, κ is an adjustable parameter and is unchanged under tortoise transformation. The parameter v_0 is an arbitrary constant. From formula (11), we can deduce some useful relations for the derivatives as follows:

$$\frac{\partial}{\partial r} = \left[1 + \frac{1}{2\kappa(r - r_H)}\right]\frac{\partial}{\partial r_*}\,, \quad \frac{\partial}{\partial v} = \frac{\partial}{\partial v_*} - \frac{r_{H,v}}{2\kappa(r - r_H)}\frac{\partial}{\partial r_*}\,.$$

Now let us consider the asymptotic behavior of R_1, R_2 near the event horizon. Under the transformation (11), Eq. (8) can be reduced to the following limiting form near the event horizon

$$\frac{\partial}{\partial r_*}R_1 = 0\,, \qquad 2r_H^2\left[G(r_H) - r_{H,v}\right]\frac{\partial}{\partial r_*}R_2 = 0\,, \tag{12}$$

after being taken the $r \to r_H(v_0)$ and $v \to v_0$ limits.

From Eq. (12), we know that R_1 is independent of r_* and regular on the event horizon. Thus the existence condition of a non-trial solution of R_2 is (as for $r_H \neq 0$)

$$2G(r_H) - 2r_{H,v} = 0\,. \tag{13}$$

which can determine the location of horizon. The event horizon equation (13) can be inferred from the null hypersurface condition, $g^{ij}\partial_i F \partial_j F = 0$, and $F(v,r) = 0$, namely $r = r(v)$. A similar procedure applying to the null surface equation

$$2\frac{\partial}{\partial v}F\frac{\partial}{\partial r}F + 2G(\frac{\partial}{\partial r}F)^2 + \frac{1}{r^2}(\frac{\partial}{\partial \theta}F)^2 + \frac{1}{r^2\sin^2\theta}(\frac{\partial}{\partial \varphi}F)^2 = 0\,, \tag{14}$$

leads to an equation

$$\Big[2G(r_H) - 2r_{H,v}\Big](\frac{\partial}{\partial r_*}F)^2 = 0\,, \tag{15}$$

resulting in the same event horizon equation due to the vanishing of the coefficients in the square bracket. As r_H depends on time v, the location of the event horizon and the shape of the black hole change with time.

4. Hawking Temperature

To investigate the Hawking radiation of spin-1/2 particles, one need consider the behavior of the second-order form of Dirac equation near the event horizon. A straightforward calculation gives the second-order radial equation

$$\begin{aligned} 2r^2\left(G\frac{\partial^2}{\partial r^2} + \frac{\partial^2}{\partial v \partial r}\right)R_1 &+ (r^2G)_{,r}\frac{\partial}{\partial r}R_1 \\ &- (\lambda^2 + \mu_0^2 r^2)R_1 = -2i\mu_0 r^2 G R_2\,, \end{aligned} \tag{16}$$

$$\begin{aligned} 2r^2\left(G\frac{\partial^2}{\partial r^2} + \frac{\partial^2}{\partial v \partial r}\right)R_2 &+ 3(r^2G)_{,r}\frac{\partial}{\partial r}R_2 + 4r\frac{\partial}{\partial v}R_2 \\ &+ \left[(r^2G)_{,rr} - (\lambda^2 + \mu_0^2 r^2)\right]R_2 = 2i\mu_0 R_1\,. \end{aligned} \tag{17}$$

Given the GTCT in Eq. (11) and after some calculations, the limiting form of Eqs. (16,17), when r approaches $r_H(v_0, \theta_0)$ and v goes to v_0, reads

$$\left[\frac{A}{2\kappa} + 2G(r_H)\right]\frac{\partial^2}{\partial r_*^2}R_1 + 2\frac{\partial^2}{\partial r_* \partial v_*}R_1 = 0\,, \tag{18}$$

and

$$\left[\frac{A}{2\kappa} + 2G(r_H)\right]\frac{\partial^2}{\partial r_*^2}R_2 + 2\frac{\partial^2}{\partial r_*\partial v_*}R_2 + \left[-A + 3G_{,r}(r_H) + \frac{2G(r_H)}{r_H}\right]\frac{\partial}{\partial r_*}R_2 = 0\,. \tag{19}$$

where we have used relations $2G(r_H) = 2r_{H,v}$ and $\frac{\partial}{\partial r_*}R_1 = 0$.

With the aid of the event horizon equation (13), namely, $2G(r_H) = 2r_{H,v}$, we know that the coefficient A is an infinite limit of 0/0 type. By use of the L' Hôspital rule, we get the following result

$$A = \lim_{r\to r_H(v_0)} \frac{2(G - r_{H,v})}{r - r_H} = 2G_{,r}(r_H)\,. \tag{20}$$

Now let us select the adjustable parameter κ in Eqs. (18,19) such that

$$\frac{A}{2\kappa} + 2G(r_H) = \frac{G_{,r}(r_H)}{\kappa} + 2r_{H,v} \equiv 1\,, \tag{21}$$

which gives the temperature of the horizon

$$\kappa = \frac{G_{,r}(r_H)}{1 - 2G(r_H)} = \frac{G_{,r}(r_H)}{1 - 2r_{H,v}}\,. \tag{22}$$

With such a parameter adjustment, we can reduce Eqs. (18,19) to

$$\frac{\partial^2}{\partial r_*^2}R_1 + 2\frac{\partial^2}{\partial r_*\partial v_*}R_1 = 0\,, \qquad \frac{\partial}{\partial r_*}R_1 = 0\,, \tag{23}$$

$$\begin{aligned}\frac{\partial^2}{\partial r_*^2}R_2 &+ 2\frac{\partial^2}{\partial r_*\partial v_*}R_2 + \left[G_{,r}(r_H) + \frac{2G(r_H)}{r_H}\right]\frac{\partial}{\partial r_*}R_2 \\ &= \frac{\partial^2}{\partial r_*^2}R_2 + 2\frac{\partial^2}{\partial r_*\partial v_*}R_2 + 2C\frac{\partial}{\partial r_*}R_2 = 0\,.\end{aligned} \tag{24}$$

where C will be regarded as a finite real constant,

$$C = \frac{1}{2}G_{,r}(r_H) + \frac{r_{H,v}}{r_H}\,.$$

Eqs. (23,24) are standard wave equations near the horizon, which can be separated by variables as the following section.

5. Thermal Radiation Spectrum

From Eq. (23), we know that R_1 is a constant on the event horizon. The solution $R_1 = R_{10}e^{-i\omega v_*}$ means that Hawking radiation does not exist for P_1, Q_2.

Now separating variables to Eq. (24) as $R_2 = R_2(r_*)e^{-i\omega v_*}$ and substituting this into equation (24), one gets

$$R_2'' = 2(i\omega - C)R_2'\,, \tag{25}$$

The solution is

$$R_2 = R_{21} e^{2(i\omega - C)r_*} + R_{20} \,. \tag{26}$$

The ingoing wave and the outgoing wave to Eq. (24) are

$$\begin{aligned} R_2^{\rm in} &= e^{-i\omega v_*} \,, \\ R_2^{\rm out} &= e^{-i\omega v_*} e^{2(i\omega - C)r_*} \,, \qquad (r > r_H) \,. \end{aligned} \tag{27}$$

Near the event horizon, we have

$$r_* \sim \frac{1}{2\kappa} \ln(r - r_H) \,.$$

Clearly, the outgoing wave $R_2^{\rm out}(r > r_H)$ is not analytic at the event horizon $r = r_H$, but can be analytically extended from the outside of the hole into the inside of the hole through the lower complex r-plane

$$(r - r_H) \rightarrow (r_H - r) e^{-i\pi}$$

to

$$\widetilde{R_2^{\rm out}} = e^{-i\omega v_*} e^{2(i\omega - C)r_*} e^{i\pi C/\kappa} e^{\pi\omega/\kappa} \,, \qquad (r < r_H) \,. \tag{28}$$

So the relative scattering probability of the outgoing wave at the horizon is easily obtained

$$\left| \frac{R_2^{\rm out}}{\widetilde{R_2^{\rm out}}} \right|^2 = e^{-2\pi\omega/\kappa} \,. \tag{29}$$

According to the method of Damour-Ruffini-Sannan's,[11] the thermal radiation Fermionic spectrum of Dirac particles from the event horizon of the hole is given by

$$\langle \mathcal{N}_\omega \rangle = \frac{1}{e^{\omega/T_H} + 1} \,, \tag{30}$$

with the Hawking temperature $T_H = \frac{\kappa}{2\pi}$

$$T_H = \frac{\kappa}{2\pi} = \frac{1}{4\pi r_H} \cdot \frac{M r_H - \Lambda r_H^4/3}{M r_H - \Lambda r_H^4/6} \,. \tag{31}$$

It follows that the temperature depends on the time, because it is determined by the surface gravity κ, a function of v.

6. Hawking Radiation of Klein-Gordon Particles

To compare the thermal radiation spectrum of electrons with that of scalar particles, we are now in a position to investigate the Hawking radiation of Klein-Gordon fields. The Klein-Gordon equation $(\Box - \mu^2)\Phi = 0$ for scalar particles with

mass μ in the Vaidya-type space-time (1) can be separated by $\Phi = R(r)Y_{\ell m}(\theta, \varphi)$ into a radial equation as

$$2r^2\left(G\frac{\partial^2}{\partial r^2} + \frac{\partial^2}{\partial v \partial r}\right)R + 2(r^2 G)_{,r}\frac{\partial}{\partial r}R \\ + 2r\frac{\partial}{\partial v}R - [\ell(\ell+1) + \mu^2 r^2]R = 0\,. \tag{32}$$

The angular part $Y_{\ell m}(\theta, \varphi)$ is the common spherical harmonic function.

Application of a similar prescription as done before to Eq. (32) yields

$$\left[\frac{A}{2\kappa} + 2G(r_H)\right]\frac{\partial^2}{\partial r_*^2}R + 2\frac{\partial^2}{\partial r_* \partial v_*}R \\ + \left[-A + 2G_{,r}(r_H) + \frac{2G(r_H)}{r_H}\right]\frac{\partial}{\partial r_*}R = 0\,. \tag{33}$$

Substituting A into Eq. (33) and making the above adjustment of the parameter κ, we can reduce the radial part of scalar wave equation to a standard form near the horizon

$$\frac{\partial^2}{\partial r_*^2}R + 2\frac{\partial^2}{\partial r_* \partial v_*}R + \frac{2G(r_H)}{r_H}\frac{\partial}{\partial r_*}R \\ = \frac{\partial^2}{\partial r_*^2}R + 2\frac{\partial^2}{\partial r_* \partial v_*}R + 2C\frac{\partial}{\partial r_*}R = 0\,, \tag{34}$$

where $C = \frac{r_{H,v}}{r_H}$.

Following the measure as done in the preceding section, one can easily derive from Eq. (34) the thermal radiation Bosonic spectrum of Klein-Gordon particles from the event horizon of a Vaidya-type black hole

$$\langle \mathcal{N}_\omega \rangle = \frac{1}{e^{\omega/T_H} - 1}\,, \tag{35}$$

The black body radiation spectra (30, 35) demonstrate that no quantum ergosphere effect can appear in the thermal radiation spectrum of electrons. The difference between both spectra lies only in the distribution factors due to differnt spin statistics.

7. Conclusions

Equations (13) and (22) give the location and the temperature of event horizon, which depend on the advanced time v. They are just the same as that obtained in the discussion on thermal radiation of Klein-Gordon particles in the same space-time. Eqs. (30,35) show, respectively, the thermal radiation spectrum of particles with spin-$1/2, 0$ in a Vaidya-type black hole. These results coincide with others. From the thermal spectrum (30) of Dirac particles, we know that there is not any new interaction energy in a Vaidya-type space-time. This manifests that there is no new quantum thermal effect called by Li[3,4] in all spherically symmetric black holes.

In conclusion, we have studied the Hawking radiation of a Vaidya-type black hole whose mass changes with time. Our results are consistent with others. We have dealt with the asymptotic behavior of the separated Dirac equation near the event horizon, not only its first-order form but also its second-order form. We find that the limiting form of its first-order form puts very strong restrict on the Hawking radiation, that is, not all components of Dirac spinors but P_2, Q_1 display the property of thermal radiation. The asymmetry of Hawking radiation with respect to the four-component Dirac spinors probably originate from the asymmetry of space-times in the advanced Eddington-Finkelstein coordinate. This point has not been revealed previously.

In addition, our analysis demonstrates that there was no new quantum ergosphere effect in a Vaidya-type space-time as declared by Li.[3,4] This conclusion holds true in any spherically symmetric black hole whether it is static or non-static.

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